1. Real Numbers

Examples

Example : 1 Consider the numbers 4^n, where n is a natural number. Check whether there is any value of n for which 4^n ends with the digit zero.

Example : 2 Find the LCM and HCF of 6 and 20 by the prime factorisation method.

Example : 3 Find the HCF of 96 and 404 by the prime factorisation method. Hence, find their LCM.

Example : 4 Find the HCF and LCM of 6, 72 and 120, using the prime factorisation method.

Example : 5 Prove that $\sqrt{3}$ is irrational.

Example : 6 Show that $5 - \sqrt{3}$ is irrational.

Example : 7 Show that $3\sqrt{2}$ is irrational.

1. Real Numbers

Exercise : 1.1

1. Express each number as a product of its prime factors:

(i) 140

1. Express each number as a product of its prime factors:

(ii) 156

1. Express each number as a product of its prime factors:

(iii) 3825

1. Express each number as a product of its prime factors:

(iv) 5005

1. Express each number as a product of its prime factors:

(v) 7429

2. Find the LCM and HCF of the following pairs of integers and verify that LCM × HCF = product of the two numbers.

(i) 26 and 91

2. Find the LCM and HCF of the following pairs of integers and verify that LCM × HCF = product of the two numbers.

(ii) 510 and 92

2. Find the LCM and HCF of the following pairs of integers and verify that LCM × HCF = product of the two numbers.

(iii) 336 and 54

3. Find the LCM and HCF of the following integers by applying the prime factorisation method.

(i) 12, 15 and 21

3. Find the LCM and HCF of the following integers by applying the prime factorisation method.

(ii) 17, 23 and 29

3. Find the LCM and HCF of the following integers by applying the prime factorisation method.

(iii) 8, 9 and 25

4. Given that HCF (306, 657) = 9, find LCM (306, 657).

5. Check whether 6^n can end with the digit 0 for any natural number n.

6. Explain why $7 \times 11 \times 13 + 13$ and $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5$ are composite numbers.

7. There is a circular path around a sports field. Sonia takes 18 minutes to drive one round of the field, while Ravi takes 12 minutes for the same. Suppose they both start at the same point and at the same time, and go in the same direction. After how many minutes will they meet again at the starting point?

1. Real Numbers

Exercise : 1.2

1. Prove that $\sqrt{5}$ is irrational.

2. Prove that $3 + 2\sqrt{5}$ is irrational.

3. Prove that the following are irrationals : (i) $\frac{1}{\sqrt{2}}$

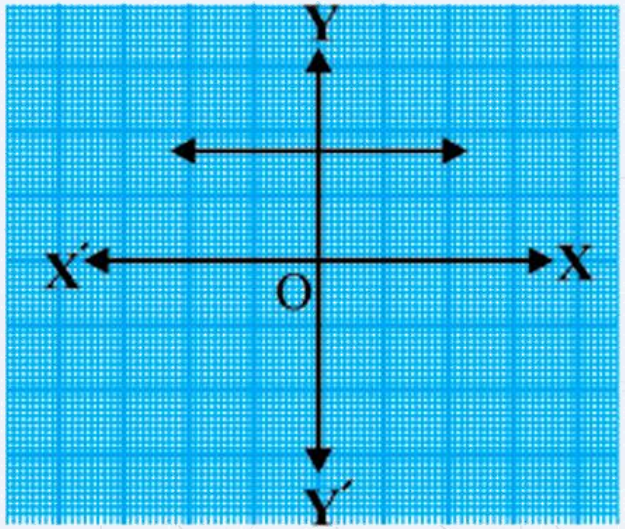
3. Prove that the following are irrationals : (ii) $7\sqrt{5}$

3. Prove that the following are irrationals : (iii) $6 + \sqrt{2}$

2. Polynomials

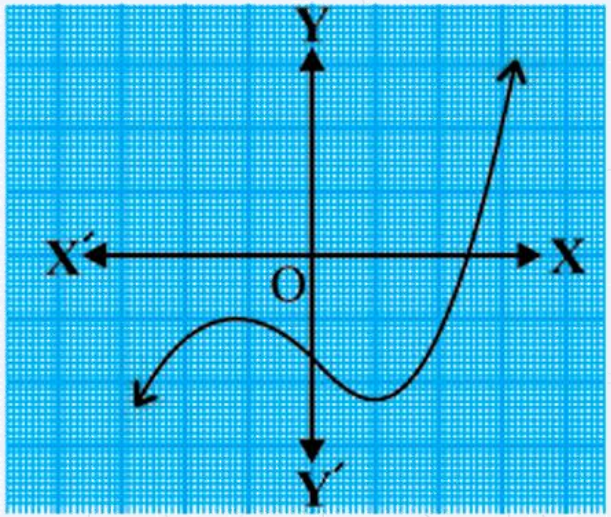
Examples

Example 1 : Look at the graphs in Fig. 2.9 given below. Each is the graph of $y = p(x)$, where $p(x)$ is a polynomial. For each of the graphs, find the number of zeroes of $p(x)$.



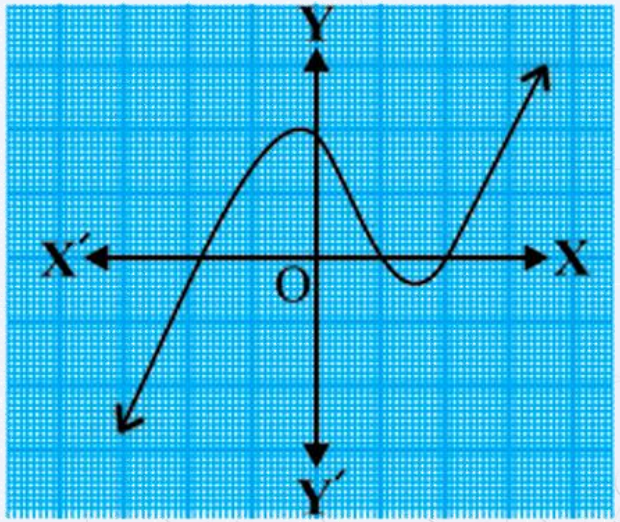
(i)

Example 1 : Look at the graphs in Fig. 2.9 given below. Each is the graph of $y = p(x)$, where $p(x)$ is a polynomial. For each of the graphs, find the number of zeroes of $p(x)$.



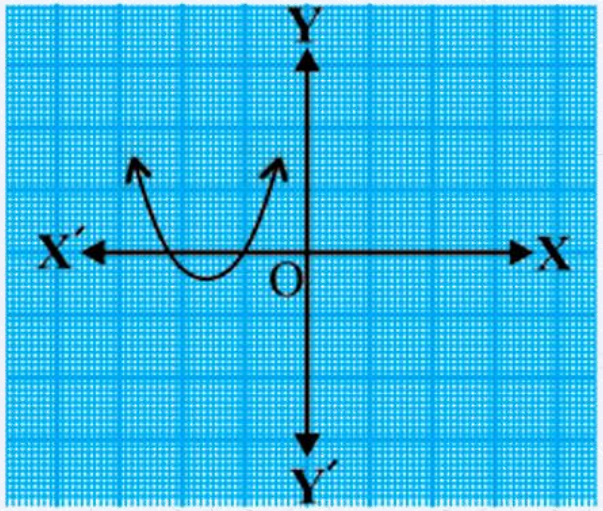
(ii)

Example 1 : Look at the graphs in Fig. 2.9 given below. Each is the graph of $y = p(x)$, where $p(x)$ is a polynomial. For each of the graphs, find the number of zeroes of $p(x)$.



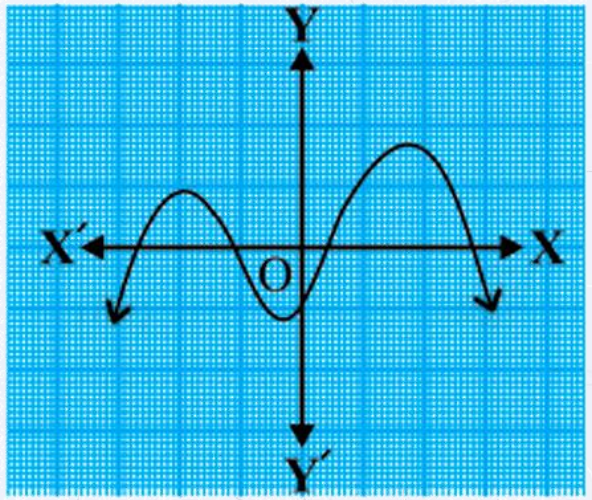
(iii)

Example 1 : Look at the graphs in Fig. 2.9 given below. Each is the graph of $y = p(x)$, where $p(x)$ is a polynomial. For each of the graphs, find the number of zeroes of $p(x)$.



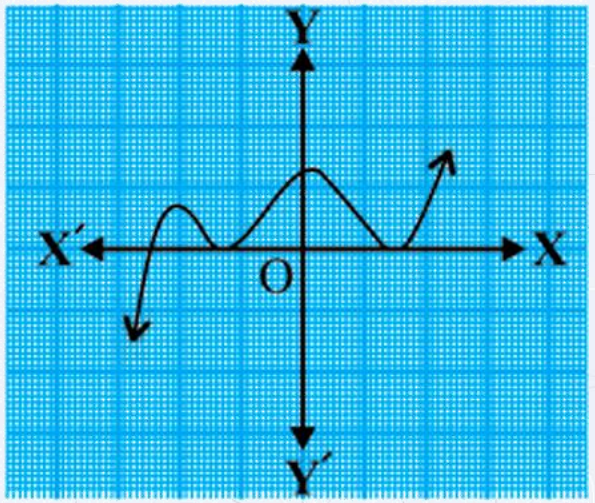
(iv)

Example 1 : Look at the graphs in Fig. 2.9 given below. Each is the graph of $y = p(x)$, where $p(x)$ is a polynomial. For each of the graphs, find the number of zeroes of $p(x)$.



(v)

Example 1 : Look at the graphs in Fig. 2.9 given below. Each is the graph of $y = p(x)$, where $p(x)$ is a polynomial. For each of the graphs, find the number of zeroes of $p(x)$.



(vi)

Example 2 : Find the zeroes of the quadratic polynomial $x^2 + 7x + 10$, and verify the relationship between the zeroes and the coefficients.

Example 3 : Find the zeroes of the polynomial $x^2 - 3$ and verify the relationship between the zeroes and the coefficients.

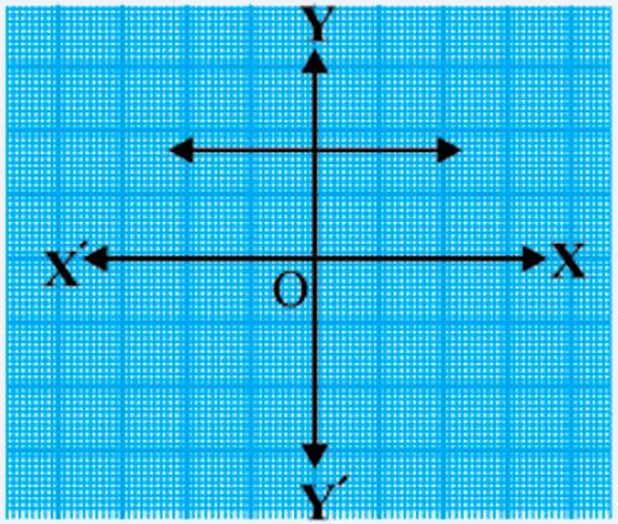
Example 4 : Find a quadratic polynomial, the sum and product of whose zeroes are -3 and 2, respectively.

Example 5* : Verify that $3, -1, -\frac{1}{3}$ are the zeroes of the cubic polynomial $p(x) = 3x^3 - 5x^2 - 11x - 3$, and then verify the relationship between the zeroes and the coefficients.

2. Polynomials

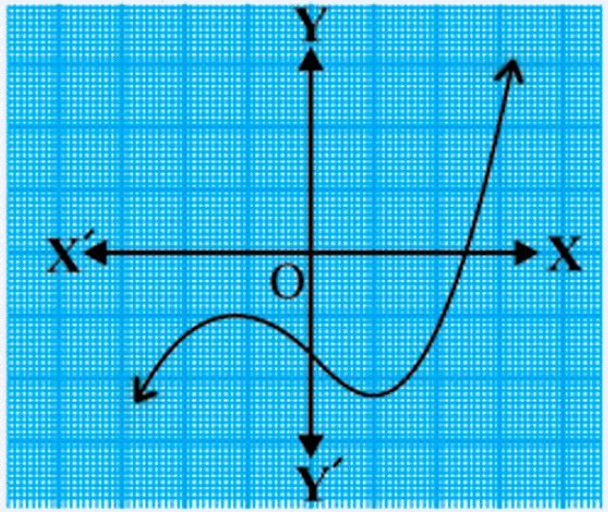
Exercise : 2.1

1. The graphs of $y = p(x)$ are given in Fig. 2.10 below, for some polynomials $p(x)$. Find the number of zeroes of $p(x)$, in each case.



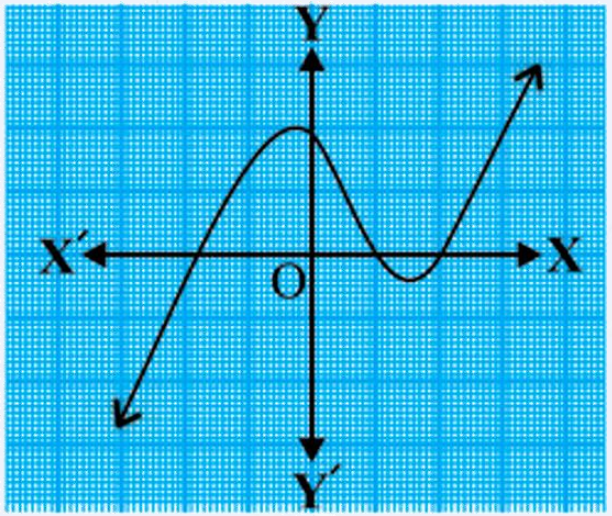
(i)

1. The graphs of $y = p(x)$ are given in Fig. 2.10 below, for some polynomials $p(x)$. Find the number of zeroes of $p(x)$, in each case.



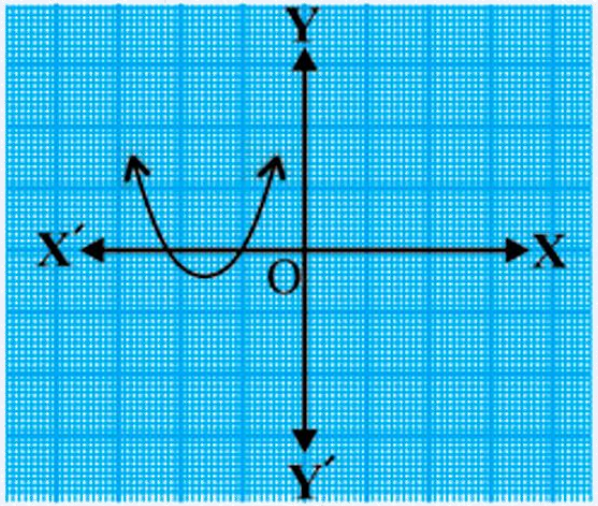
(ii)

1. The graphs of $y = p(x)$ are given in Fig. 2.10 below, for some polynomials $p(x)$. Find the number of zeroes of $p(x)$, in each case.



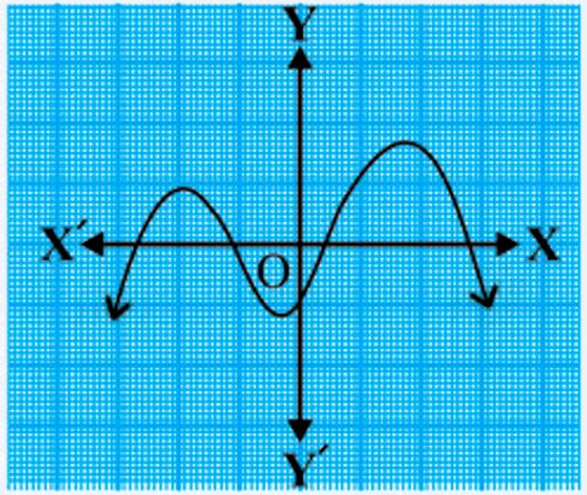
(iii)

1. The graphs of $y = p(x)$ are given in Fig. 2.10 below, for some polynomials $p(x)$. Find the number of zeroes of $p(x)$, in each case.



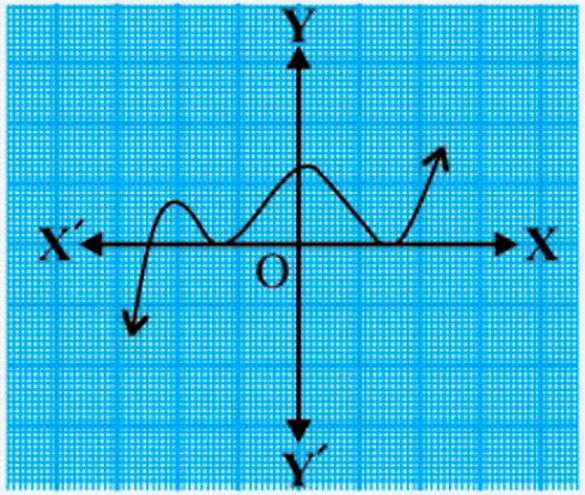
(iv)

1. The graphs of $y = p(x)$ are given in Fig. 2.10 below, for some polynomials $p(x)$. Find the number of zeroes of $p(x)$, in each case.



(v)

1. The graphs of $y = p(x)$ are given in Fig. 2.10 below, for some polynomials $p(x)$. Find the number of zeroes of $p(x)$, in each case.



(vi)

2. Polynomials

Exercise : 2.2

1. Find the zeroes of the following quadratic polynomials and verify the relationship between the zeroes and the coefficients.

(i) $x^2 - 2x - 8$

1. Find the zeroes of the following quadratic polynomials and verify the relationship between the zeroes and the coefficients.

(ii) $4s^2 - 4s + 1$

1. Find the zeroes of the following quadratic polynomials and verify the relationship between the zeroes and the coefficients.

(iii) $6x^2 - 3 - 7x$

1. Find the zeroes of the following quadratic polynomials and verify the relationship between the zeroes and the coefficients.

(iv) $4u^2 + 8u$

1. Find the zeroes of the following quadratic polynomials and verify the relationship between the zeroes and the coefficients.

(v) $t^2 - 15$

1. Find the zeroes of the following quadratic polynomials and verify the relationship between the zeroes and the coefficients.

(vi) $3x^2 - x - 4$

2. Find a quadratic polynomial each with the given numbers as the sum and product of its zeroes respectively.

(i) $\frac{1}{4}, -1$

2. Find a quadratic polynomial each with the given numbers as the sum and product of its zeroes respectively.

(ii) $\sqrt{2}, \frac{1}{3}$

2. Find a quadratic polynomial each with the given numbers as the sum and product of its zeroes respectively.

(iii) $0, \sqrt{5}$

2. Find a quadratic polynomial each with the given numbers as the sum and product of its zeroes respectively.

(iv) 1, 1

2. Find a quadratic polynomial each with the given numbers as the sum and product of its zeroes respectively.

(v) $-\frac{1}{4}, \frac{1}{4}$

2. Find a quadratic polynomial each with the given numbers as the sum and product of its zeroes respectively.

(vi) 4, 1

3. Pair of Linear Equations in Two Variables

Examples

1. Check graphically whether the pair of equations $x + 3y = 6$ and $2x - 3y = 12$ is consistent. If so, solve them graphically.

2. Graphically, find whether the following pair of equations has no solution, unique solution or infinitely many solutions :

$5x - 8y + 1 = 0$

$3x - \frac{24}{5}y + \frac{3}{5} = 0$

3. Champa went to a 'Sale' to purchase some pants and skirts. When her friends asked her how many of each she had bought, she answered, "The number of skirts is two less than twice the number of pants purchased. Also, the number of skirts is four less than four times the number of pants she purchased". Help her friends to find how many pants and skirts Champa bought.

4. Solve the following pair of equations by substitution method :

$7x - 15y = 2$ ....(i)

$x + 2y = 3$ ....(ii)

5. Aftab tells his daughter, "seven years ago, I was seven times as old as you were then. Also three years from now, I shall be three times as old as you will be." (Isn't this interesting ?) Represent this situation algebraically and graphically by the method of substitution.

6. In a shop the cost of 2 pencils and 3 erasers is ₹ 9 and the cost of 4 Pencils and 6 erasers is ₹ 18. Find the cost of each pencil and each eraser by the method of substitution.

7. Two rails are represented by the equations $x + 2y - 4 = 0$ and $2x + 4y - 12 = 0$. Will the rails cross ?

8. The ratio of incomes of two persons is 9 : 7 and the ratio of their expenditures is 4 : 3. If each of them manages to solve ₹ 2000 per month, find their monthly incomes.

9. Use elimination method to find all possible solutions of the following pair of linear equations :

$2x + 3y = 8$

$4x + 6y = 7$

10. The sum of a two-digit number and the number obtained by reversing the digits is 66. If the digits of the number differ by 2, find the number. How many such numbers are there ?

3. Pair of Linear Equations in Two Variables

Exercise : 3.1

1. Form the pair of linear equations in the following problems, and find their solutions graphically.

(i) 10 students of Class X took part in a Mathematics quiz. If the number of girls is 4 more than the number of boys, find the number of boys and girls who took part in the quiz.

1. Form the pair of linear equations in the following problems, and find their solutions graphically.

(ii) 5 pencils and 7 pens together cost ₹ 50, whereas 7 pencils and 5 pens together cost ₹ 46. Find the cost of one pencil and that of one pen.

2. On comparing the ratios $\frac{a_1}{a_2}, \frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$, find out whether the lines representing the following pairs of linear equations intersect at a point, are parallel or coincident:

(i) $5x - 4y + 8 = 0$

$7x + 6y - 9 = 0$

2. On comparing the ratios $\frac{a_1}{a_2}, \frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$, find out whether the lines representing the following pairs of linear equations intersect at a point, are parallel or coincident:

(ii) $9x + 3y + 12 = 0$

$18x + 6y + 24 = 0$

2. On comparing the ratios $\frac{a_1}{a_2}, \frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$, find out whether the lines representing the following pairs of linear equations intersect at a point, are parallel or coincident:

(iii) $6x - 3y + 10 = 0$

$2x - y + 9 = 0$

3. On comparing the ratios $\frac{a_1}{a_2}, \frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$, find out whether the following pair of linear equations are consistent, or inconsistent.

(i) $3x + 2y = 5$; $2x - 3y = 7$

3. On comparing the ratios $\frac{a_1}{a_2}, \frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$, find out whether the following pair of linear equations are consistent, or inconsistent.

(ii) $2x - 3y = 8$; $4x - 6y = 9$

3. On comparing the ratios $\frac{a_1}{a_2}, \frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$, find out whether the following pair of linear equations are consistent, or inconsistent.

(iii) $\frac{3}{2}x + \frac{5}{3}y = 7 \; ; \; 9x - 10y = 14$

3. On comparing the ratios $\frac{a_1}{a_2}, \frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$, find out whether the following pair of linear equations are consistent, or inconsistent.

(iv) $5x - 3y = 11\ ;\ -10x + 6y = -22$

3. On comparing the ratios $\frac{a_1}{a_2}, \frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$, find out whether the following pair of linear equations are consistent, or inconsistent.

(v) $\frac{4}{3}x + 2y = 8$; $2x + 3y = 12$

4. Which of the following pairs of linear equations are consistent/inconsistent? If consistent, obtain the solution graphically:

(i) $x + y = 5,$ $\quad 2x + 2y = 10$

4. Which of the following pairs of linear equations are consistent/inconsistent? If consistent, obtain the solution graphically:

(ii) $x - y = 8$, $3x - 3y = 16$

4. Which of the following pairs of linear equations are consistent/inconsistent? If consistent, obtain the solution graphically:

(iii) $2x + y - 6 = 0, \quad 4x - 2y - 4 = 0$

4. Which of the following pairs of linear equations are consistent/inconsistent? If consistent, obtain the solution graphically:

(iv) $2x - 2y - 2 = 0, \quad 4x - 4y - 5 = 0$

5. Half the perimeter of a rectangular garden, whose length is 4 m more than its width, is 36 m. Find the dimensions of the garden.

6. Given the linear equation $2x + 3y - 8 = 0$, write another linear equation in two variables such that the geometrical representation of the pair so formed is:

(i) intersecting lines

6. Given the linear equation $2x + 3y - 8 = 0$, write another linear equation in two variables such that the geometrical representation of the pair so formed is:

(ii) parallel lines

6. Given the linear equation $2x + 3y - 8 = 0$, write another linear equation in two variables such that the geometrical representation of the pair so formed is:

(iii) coincident lines

7. Draw the graphs of the equations $x - y + 1 = 0$ and $3x + 2y - 12 = 0$. Determine the coordinates of the vertices of the triangle formed by these lines and the x-axis, and shade the triangular region.

3. Pair of Linear Equations in Two Variables

Exercise : 3.2

1. Solve the following pair of linear equations by the substitution method.

(i) $x + y = 14$

$x - y = 4$

1. Solve the following pair of linear equations by the substitution method.

(ii) $s - t = 3$

$$\frac{s}{3} + \frac{t}{2} = 6$$

1. Solve the following pair of linear equations by the substitution method.

(iii) $3x - y = 3$
$9x - 3y = 9$

1. Solve the following pair of linear equations by the substitution method.

(iv) $0.2x + 0.3y = 1.3$
$0.4x + 0.5y = 2.3$

1. Solve the following pair of linear equations by the substitution method.

(v) $\sqrt{2}\,x+\sqrt{3}\,y=0$

$\sqrt{3}\,x-\sqrt{8}\,y=0$

1. Solve the following pair of linear equations by the substitution method.

(vi) $\frac{3x}{2} - \frac{5y}{3} = -2$

$\frac{x}{3} + \frac{y}{2} = \frac{13}{6}$

2. Solve $2x + 3y = 11$ and $2x - 4y = -24$ and hence find the value of 'm' for which $y = mx + 3$.

3. Form the pair of linear equations for the following problems and find their solution by substitution method.

(i) The difference between two numbers is 26 and one number is three times the other. Find them.

3. Form the pair of linear equations for the following problems and find their solution by substitution method.

(ii) The larger of two supplementary angles exceeds the smaller by 18 degrees. Find them.

3. Form the pair of linear equations for the following problems and find their solution by substitution method.

(iii) The coach of a cricket team buys 7 bats and 6 balls for ₹ 3800. Later, she buys 3 bats and 5 balls for ₹ 1750. Find the cost of each bat and each ball.

3. Form the pair of linear equations for the following problems and find their solution by substitution method.

(iv) The taxi charges in a city consist of a fixed charge together with the charge for the distance covered. For a distance of 10 km, the charge paid is ₹ 105 and for a journey of 15 km, the charge paid is ₹ 155. What are the fixed charges and the charge per km? How much does a person have to pay for travelling a distance of 25 km?

3. Form the pair of linear equations for the following problems and find their solution by substitution method.

(v) A fraction becomes $\frac{9}{11}$, if 2 is added to both the numerator and the denominator. If, 3 is added to both the numerator and the denominator it becomes $\frac{5}{6}$. Find the fraction.

3. Form the pair of linear equations for the following problems and find their solution by substitution method.

(vi) Five years hence, the age of Jacob will be three times that of his son. Five years ago, Jacob's age was seven times that of his son. What are their present ages?

3. Pair of Linear Equations in Two Variables

Exercise : 3.3

1. Solve the following pair of linear equations by the elimination method and the substitution method :

(i) $x + y = 5$ and $2x - 3y = 4$

1. Solve the following pair of linear equations by the elimination method and the substitution method :

(ii) $3x + 4y = 10$ and $2x - 2y = 2$

1. Solve the following pair of linear equations by the elimination method and the substitution method :

(iii) $3x - 5y - 4 = 0$ and $9x = 2y + 7$

1. Solve the following pair of linear equations by the elimination method and the substitution method :

(iv) $\frac{x}{2} + \frac{2y}{3} = -1$ and $x - \frac{y}{3} = 3$

2. Form the pair of linear equations in the following problems, and find their solutions (if they exist) by the elimination method :

(i) If we add 1 to the numerator and subtract 1 from the denominator, a fraction reduces to 1. It becomes $\frac{1}{2}$ if we only add 1 to the denominator. What is the fraction?

2. Form the pair of linear equations in the following problems, and find their solutions (if they exist) by the elimination method :

(ii) Five years ago, Nuri was thrice as old as Sonu. Ten years later, Nuri will be twice as old as Sonu. How old are Nuri and Sonu?

2. Form the pair of linear equations in the following problems, and find their solutions (if they exist) by the elimination method :

(iii) The sum of the digits of a two-digit number is 9. Also, nine times this number is twice the number obtained by reversing the order of the digits. Find the number.

2. Form the pair of linear equations in the following problems, and find their solutions (if they exist) by the elimination method :

(iv) Meena went to a bank to withdraw ₹ 2000. She asked the cashier to give her ₹ 50 and ₹ 100 notes only. Meena got 25 notes in all. Find how many notes of ₹ 50 and ₹ 100 she received.

2. Form the pair of linear equations in the following problems, and find their solutions (if they exist) by the elimination method :

(v) A lending library has a fixed charge for the first three days and an additional charge for each day thereafter. Saritha paid ₹ 27 for a book kept for seven days, while Susy paid ₹ 21 for the book she kept for five days. Find the fixed charge and the charge for each extra day.

4. Quadratic Equations

Examples

Example 1 : Represent the following situations mathematically:

(i) John and Jivanti together have 45 marbles. Both of them lost 5 marbles each, and the product of the number of marbles they now have is 124. We would like to find out how many marbles they had to start with.

Example 1 : Represent the following situations mathematically:

(ii) A cottage industry produces a certain number of toys in a day. The cost of production of each toy (in rupees) was found to be 55 minus the number of toys produced in a day. On a particular day, the total cost of production was ₹ 750. We would like to find out the number of toys produced on that day.

Example 2 : Check whether the following are quadratic equations:

(i) $(x-2)^2+1=2x-3$

Example 2 : Check whether the following are quadratic equations:

(ii) $x(x + 1) + 8 = (x + 2)(x - 2)$

Example 2 : Check whether the following are quadratic equations:

(iii) $x\,(2x + 3) = x^2 + 1$

Example 2 : Check whether the following are quadratic equations:

(iv) $(x + 2)^3 = x^3 - 4$

Example 3 : Find the roots of the equation $2x^2 - 5x + 3 = 0$, by factorisation.

Example 4 : Find the roots of the quadratic equation $6x^2 - x - 2 = 0$.

Example 5 : Find the roots of the quadratic equation $3x^2 - 2\sqrt{6}x + 2 = 0$.

Example 6 : Find the dimensions of the prayer hall discussed in Section 4.1.

Example 7: Find the discriminant of the quadratic equation $2x^2 - 4x + 3 = 0$, and hence find the nature of its roots.

Example 8 : A pole has to be erected at a point on the boundary of a circular park of diameter 13 metres in such a way that the differences of its distances from two diametrically opposite fixed gates A and B on the boundary is 7 metres. Is it possible to do so? If yes, at what distances from the two gates should the pole be erected?

Example 9 : Find the discriminant of the equation $3x^2 - 2x + \frac{1}{3} = 0$ and hence find the nature of its roots. Find them, if they are real.

4. Quadratic Equations

Exercise : 4.1

1. Check whether the following are quadratic equations :

(i) $(x+1)^2 = 2(x-3)$

1. Check whether the following are quadratic equations :

(ii) $x^2 - 2x = (-2)(3 - x)$

1. Check whether the following are quadratic equations :

(iii) $(x-2)(x+1)=(x-1)(x+3)$

1. Check whether the following are quadratic equations :

(iv) $(x-3)(2x+1)=x(x+5)$

1. Check whether the following are quadratic equations :

(v) $(2x-1)(x-3)=(x+5)(x-1)$

1. Check whether the following are quadratic equations :

(vi) $x^2 + 3x + 1 = (x - 2)^2$

1. Check whether the following are quadratic equations :

(vii) $(x+2)^3 = 2x(x^2-1)$

1. Check whether the following are quadratic equations :

(viii) $x^3 - 4x^2 - x + 1 = (x - 2)^3$

2. Represent the following situations in the form of quadratic equations :

(i) The area of a rectangular plot is 528 m^2. The length of the plot (in metres) is one more than twice its breadth. We need to find the length and breadth of the plot.

2. Represent the following situations in the form of quadratic equations :

(ii) The product of two consecutive positive integers is 306. We need to find the integers.

2. Represent the following situations in the form of quadratic equations :

(iii) Rohan's mother is 26 years older than him. The product of their ages (in years) 3 years from now will be 360. We would like to find Rohan's present age.

2. Represent the following situations in the form of quadratic equations :

(iv) A train travels a distance of 480 km at a uniform speed. If the speed had been 8 km/h less, then it would have taken 3 hours more to cover the same distance. We need to find the speed of the train.

4. Quadratic Equations

Exercise : 4.2

1. Find the roots of the following quadratic equations by factorisation:

(i) $x^2 - 3x - 10 = 0$

1. Find the roots of the following quadratic equations by factorisation:

(ii) $2x^2 + x - 6 = 0$

1. Find the roots of the following quadratic equations by factorisation:

(iii) $\sqrt{2}\,x^2 + 7x + 5\sqrt{2} = 0$

1. Find the roots of the following quadratic equations by factorisation:

(iv) $2x^2 - x + \frac{1}{8} = 0$

1. Find the roots of the following quadratic equations by factorisation:

(v) $100x^2 - 20x + 1 = 0$

2. (i) John and Jivanti together have 45 marbles. Both of them lost 5 marbles each, and the product of the number of marbles they now have is 124. We would like to find out how many marbles they had to start with.

2. (ii) A cottage industry produces a certain number of toys in a day. The cost of production of each toy (in rupees) was found to be 55 minus the number of toys produced in a day. On a particular day, the total cost of production was ₹ 750. We would like to find out the number of toys produced on that day.

3. Find two numbers whose sum is 27 and product is 182.

4. Find two consecutive positive integers, sum of whose squares is 365.

5. The altitude of a right triangle is 7 cm less than its base. If the hypotenuse is 13 cm, find the other two sides.

6. A cottage industry produces a certain number of pottery articles in a day. It was observed on a particular day that the cost of production of each article (in rupees) was 3 more than twice the number of articles produced on that day. If the total cost of production on that day was ₹ 90, find the number of articles produced and the cost of each article.

4. Quadratic Equations

Exercise : 4.3

1. Find the nature of the roots of the following quadratic equations. If the real roots exist, find them:

(i) $2x^2 - 3x + 5 = 0$

1. Find the nature of the roots of the following quadratic equations. If the real roots exist, find them:

(ii) $3x^2 - 4\sqrt{3}\,x + 4 = 0$

1. Find the nature of the roots of the following quadratic equations. If the real roots exist, find them:

(iii) $2x^2 - 6x + 3 = 0$

2. Find the values of k for each of the following quadratic equations, so that they have two equal roots.

(i) $2x^2 + kx + 3 = 0$

2. Find the values of k for each of the following quadratic equations, so that they have two equal roots.

(ii) $kx\,(x-2)+6=0$

4. Is the following situation possible? If so, determine their present ages.
The sum of the ages of two friends is 20 years. Four years ago, the product of their ages in years was 48.

5. Is it possible to design a rectangular park of perimeter 80 m and area 400 m^2? If so, find its length and breadth.